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CCXXXVIII.

(Vol. XI.—June, 1882.)

SUB-AQUEOUS UNDERPINNING.

By A. G. MENOCAL, Member A. S. C. E.

READ AT THE ANNUAL CONVENTION, MAY 17TH, 1882.

The quay wall at the Gosport Navy Yard was built between the years 1835 and 1842. It was constructed of cut granite, and rested on a timber platform, capping foundation piles, 18 feet below the coping, and 13 feet 4 inches below the level of high water. For cross section of wall, see Fig. 1, Plate VI.

Soon after the completion of the work, the depth of water in front of the quay was found to be totally inadequate for the class of ships calling at that station, and the channel was improved by dredging. The excavation was gradually carried on, and it appears no apprehension was felt by those in charge, that the safety of the wall was involved in that operation. The removal of the mud in close proximity to the foundation

caused a gradual sliding of the material in front of and beneath the timber platform, and from around the piling, leaving the wood exposed to the attacks of the teredo, so abundant in those waters. The effects of this injudicious operation were shortly made manifest by a gradual settling of the masonry, not sufficient in amount, however, to occasion any alarm for several years. In fact, it was not until the year 1875 that a marked increase of the movement was observed. In a brief period the wall sunk several inches, and a decided outward movement was also observed. An examination of the foundations, with the assistance of divers, established the fact that both the platform and pile foundation had been partially destroyed by the teredo. The supporting power of the upper exposed ends of the front rows of piles was very greatly impaired, and the larger part of the weight of the structure rested on the two rear rows (see Fig. 1). The bond of the stone-work also contributed to the stability of the structure. With a view of arresting the settlement of the wharf, and in the hope of preventing its destruction, a row of eighteen inch square logs was about that time driven close together, and in immediate contact with the outer edge of the foundation (Fig. 1 *A*), but this expedient failed to meet the expectations, and the wall continued to settle at a rapid rate.

A Board of three Civil Engineers of the Navy, under orders from the Navy Department, examined the work in 1880, and found that the greatest settlement was about 18 inches, and that the outer face of the wall had, in some places, moved 10 inches outwards.

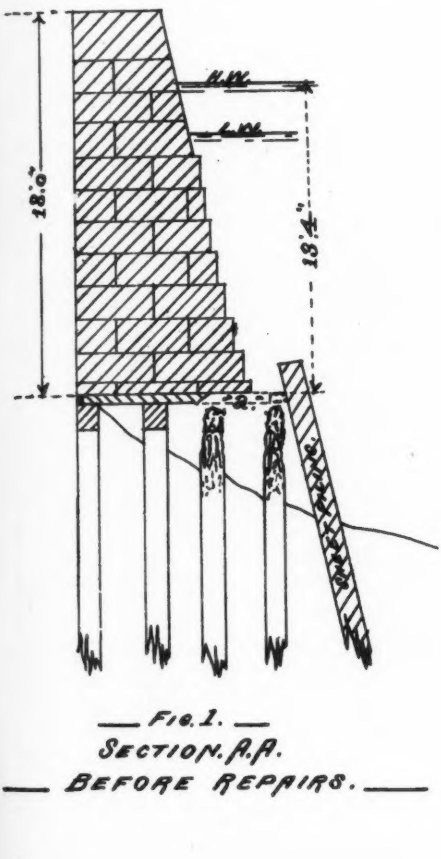
A new examination of the foundation by divers was found to be impracticable, on account of the sheet piles in front, but the information afforded by the previous examination was thought sufficient to remove all doubts as to the causes of the damage.

The sinking of the wall was evidently due to the partial destruction of the wood foundations, and the questions that naturally suggested themselves were :

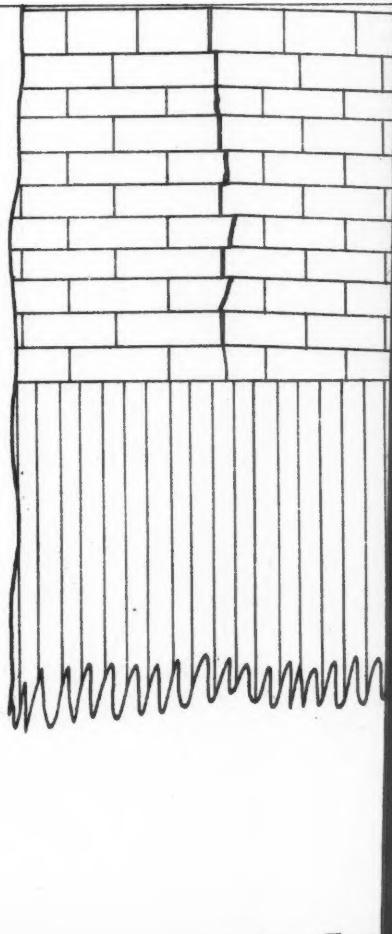
First.—Could any means be devised for permanently arresting the settlement ?

Second.—Should the structure be demolished, and a new one erected in its place on an entirely new and deeper foundation ?

The second alternative involved an expenditure far in excess of the amount available for these repairs, and additional appropriations for the same object could not be depended upon.



— FIG. 1. —
SECTION. A.A.
— BEFORE REPAIRS. —



— FRO

QUAY WALL

U.S. NAVY-YARD. NOR

SHOWING

FRONT ELEVATIONS

BEFORE & AFT

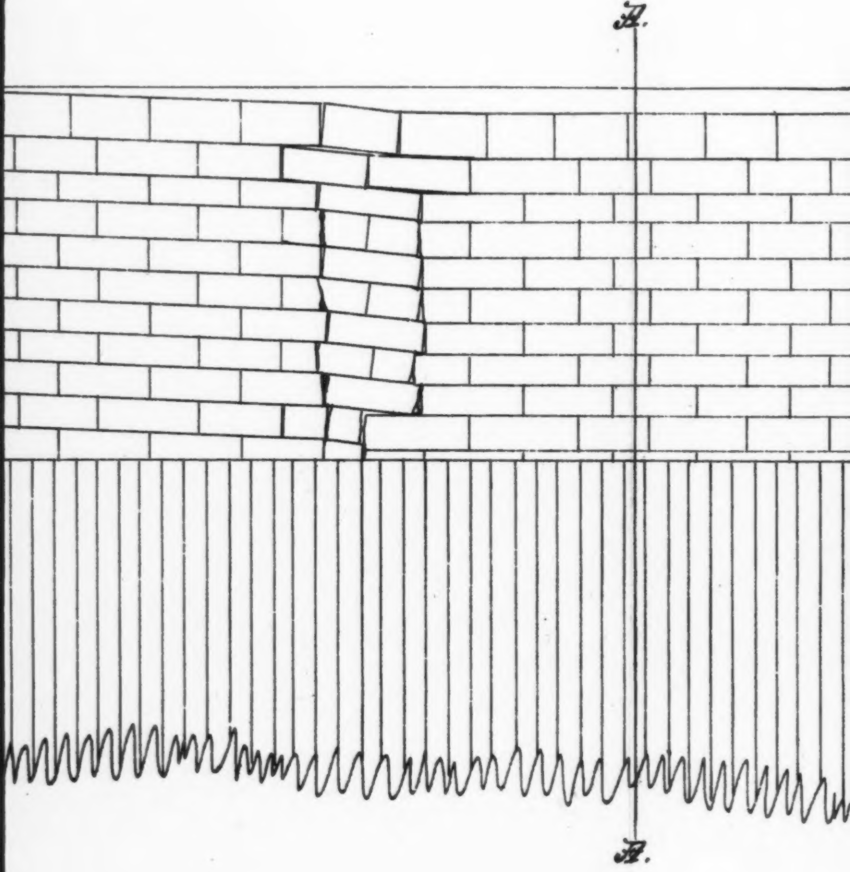


FIG. 2.

FRONT ELEVATION. BEFORE REPAIRS.

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NORFOLK. VA.

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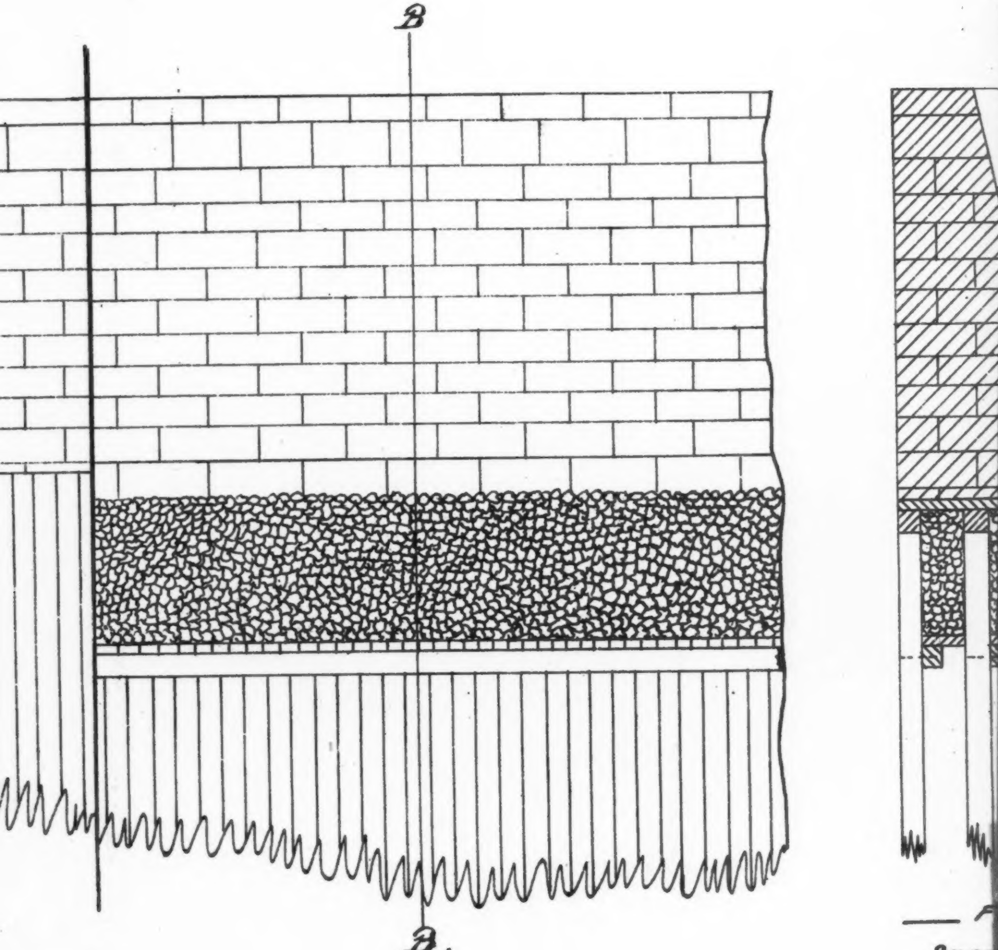
AFTER REPAIRS



2. —

— FRONT ELEVATION, AFTER REPAIRS. —

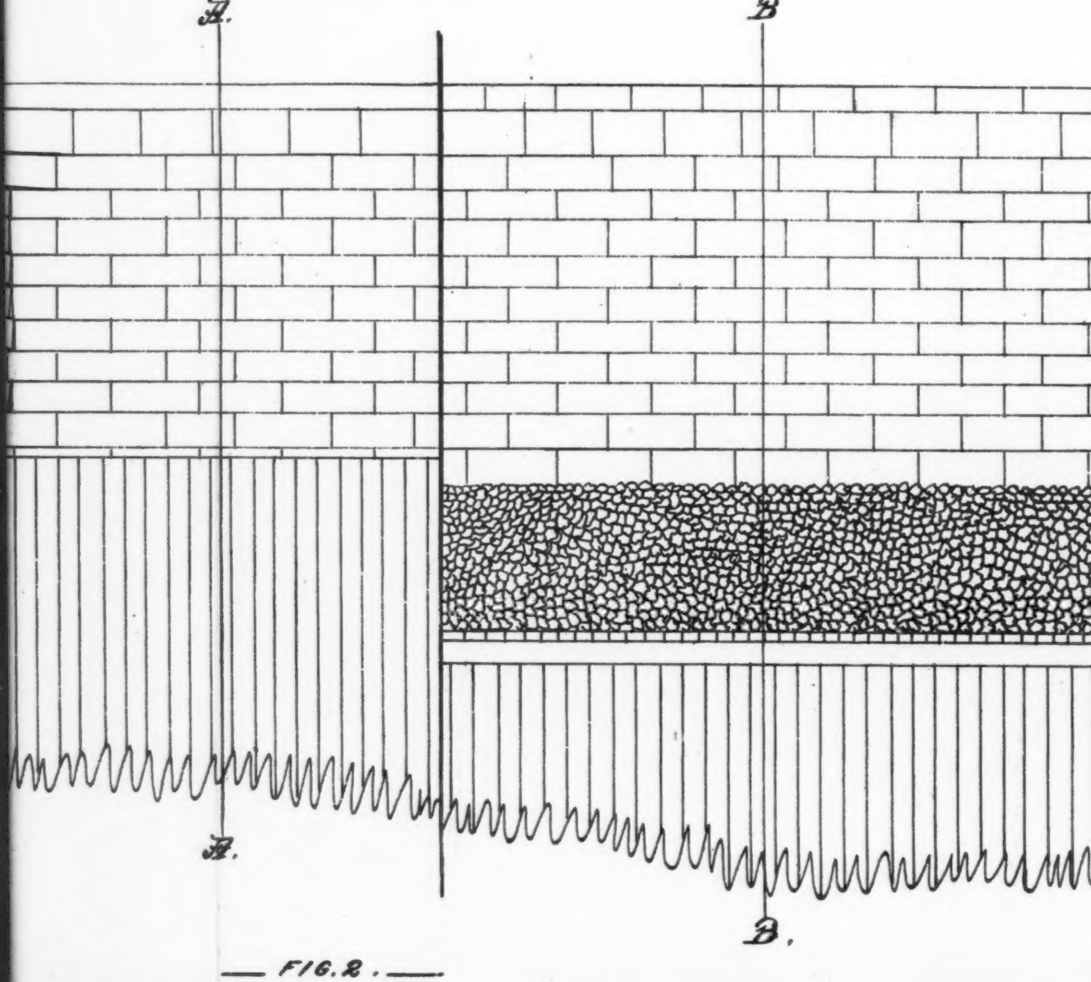
— QUAY WALL. —

U. S. NAVY-YARD, NORFOLK, VA.

SHOWING

FRONT ELEVATIONS & SECTIONS.

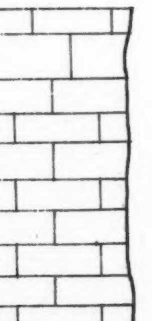
BEFORE & AFTER REPAIRS



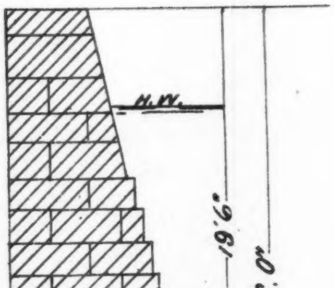
— FIG. 2. —

...RE REPAIRS. — — FRONT ELEVATION, AFTER REPAI...

PLATE VI.
TRANS. AM. SOC. CIV. ENG'R'S.
VOL. XI Nº CCXXXIX
MENOCAL ON
SUBAQUEOUS UNDERPINNING.



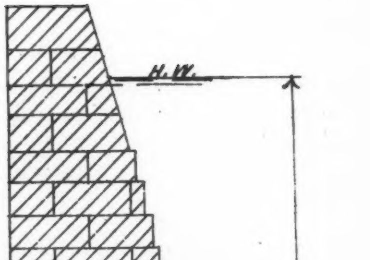
REPAIRS. —



— FIG. 3. —

SECTION. B.B.

— AFTER REPAIRS. —



— FIG. 4. —

The masonry was partially held in place by the bond of the cut-stone work, and any attempt at the removal of the material would destroy the integrity of the structure, break the bond, and quite likely result in the whole mass being precipitated to the bottom. In such a contingency the expense of the removal of the debris would be a large sum, and could not be estimated with any close approximation.

As the idea of demolishing and rebuilding the work had to be regarded as a last resort, the importance of a thorough consideration of the first proposition was, therefore, too apparent to admit of discussion.

A method of securing the desired object was submitted by the writer, who was a member of the Board, and it was adopted. The work was executed in accordance with the project submitted, which, briefly stated, was as follows:

The portion of the wall most needing repairs was a stretch of 140 feet, where the movement had been greatest. This was marked off into sections of about six feet each. Operations were commenced on the centre subdivision, where the sheet piling was first cut away; then two foundation piles in the first, and the same number in the second row, were sawed off at the level of the bottom; the remnants of the old platform were removed, and a recess was thus formed under the quay wall, some six feet wide by 6.5 feet high, and extending back 3 feet from the face of the masonry to the third row of piles.

Upon the solid timber of the piling at the bottom 12″ x 12″ stringers were bolted, and on them 6-inch yellow pine planking was secured, then two stumps of piles in the same transverse (third longitudinal) rows were cut off, and the timber capping put on, this latter covering being usually a few inches higher than the one nearest the front (see Fig. 4). The two inner rows of piles (third and fourth longitudinally) were in some places found uninjured by the worms. Stout logs were in that case firmly bolted to the piles (see Fig. 3), and the platform carried on a level to the rear. The masonry overhead, if showing signs of insecurity, was carefully shored and secured against falling. Upon the back or highest shelf of the stepped platform were then laid sacks of concrete, each containing no more than two cubic feet, the material arranged and placed so there should be very few, if any, voids between the different sacks. The concrete in contact with the stone masonry above, was thoroughly consolidated and rammed. The next lowest step was then loaded in like manner, with concrete in sacks,

and by this process a pier of hydraulic cement concrete, resting on a pile foundation, which was out of the reach of the teredo, was formed under the centre of the wall; then the subdivisions midway between the centres and the extremities of the long sections, were in like manner excavated and the wall underpinned; next the subdivisions which were nearest the centres of the four intermediate spaces were cleared out and piers of concrete built, and so the work was continued until the completed piers were but about six feet apart throughout the whole stretch, when work was carried on simultaneously on all remaining subdivisions until the gaps were closed. The cap pieces supporting the platform of the adjacent piers, were scarfed and bolted together so as to form as continuous stringers as practicable under the circumstances.

During the progress of the work, the wall was left entirely undisturbed, special care being taken to direct from it the adjacent land drainage. The filling behind the wall had become thoroughly consolidated, and exerted but little pressure on the masonry.

The estimate submitted with the project, of the cost of underpinning 140 lineal feet of the wall was as follows:

282	Piles, cut, at $4	$1 128
282	" capped, at $4	1 128
130	" cut in front row, at $2	260
2 500	Feet Yellow Pine, for caps, at 40c.	1 000
20 000	" " " " planking, at $30	600
2 240	Sq. Ft. Planking laid, at 50c.	1 120
700	Cub. Yds. Concrete, in bags, at $15	10 500
2	Sets Armor, for divers, at $1 200	2 400
20	Diving Dresses, at $50	1 000
	Tools	500
		$19 636
	Add for contingencies	5 364
	Total	$25 000

The estimated cost per lineal foot was, therefore, about $178. The estimated cost of a new wall, excluding the expense of removing the debris and otherwise preparing the ground for the construction of the new foundation, was $447 per lineal foot. The work was under the im-

mediate charge of Civil Engineer P. C. Asserson, U. S. Navy, who deserves much credit for its economic, rapid and judicious execution.

Much difficulty was at first experienced in removing from under the wall the fragments of stone and timber, which had formed with the clay and mud, a compact, stiff mass, very troublesome to move with the pick or shovel. A powerful jet of water from a hose attached to a steam pump, on the wharf, disintegrated these materials and they were readily removed to the desired depth.

Operations were begun in the month of September, 1880, and 300 feet of wall had been completed in January, 1882, but much time was lost during the winters on account of the cold.

The actual cost, including labor, material and tools, has been $125.10 per lineal foot, or about $53 less per foot than the estimate.

During the construction of the work a settling of only $7\frac{7}{8}$ inches took place, and since its completion no subsidence has occurred.

The present depth of water at the foot of the wall is 21 feet six inches.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
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NOTE.—This Society is not responsible, as a body, for the facts and opinions advanced in any of its publications.

CCXXXIX.
Vol. XI.—June, 1882.

THE MEAN VELOCITY OF STREAMS FLOWING IN NATURAL CHANNELS.

By ROBERT E. McMATH, Member A. S. C. E.

READ FEBRUARY 15TH, 1882.

1. Numerous attempts have been made to discover a relation between the mean velocity of a stream and some observable element or elements; but the formulas proposed have all failed to satisfy when applied under the varying conditions met in practice. Still, a species of progress has been made, for the limitations of the problem have been one by one developed.

2. It is now generally admitted that the solution cannot be a formula with general co-efficients. Local conditions control relations, and the pertinent question, of late, has been whether conditions can be grouped

into classes or categories, or must be treated in their individual combinations. Departing from the ordinary track of discussion, I propose to separate the natural stream from the artificial channel, because absence of continuous bed slope, or gradient of bottom, is a radical distinction in the conditions of flow, and to show that the definite law of discharge over a weir is usefully applicable at any transverse section above and within the influence of a weir, dam, or shoal.

3. Of the two classes under which the formulas hitherto proposed may be arranged: the one which seeks a relation between mean and maximum velocity, after falling into disrepute, has again been brought forward by one of the latest writers as being the "best means of rapid approximation to mean velocity." This unexpected conclusion is explainable only by the fact that the Roorkee experiments were in a canal, and that sections of a uniform type, at localities where original bed slope had been preserved, were chosen for experimental sites. The general formula

$$V \text{ mean} = C. V \text{ maximum},$$

is equivalent to an assertion, that the mean ordinate of any figure bears a fixed ratio to the maximum ordinate, which is true of regular curves, *i. e.*, those whose quadrature may be expressed in simple terms, but there can be no ratio common to all classes of curves.

The transverse curve of velocity, resulting from plotting the means of all verticals, will have an outline which depends upon the transverse profile of the section. For strictly similar sections a ratio may be found which will answer reasonably well, but let the engineer clearly understand that its use is restricted to very narrow limits, requiring an exact repetition of all the essential conditions.* What the essential conditions are, and the improbability of their repetition, will appear incidentally in the course of this paper.

For rivers, and natural streams in general, dissimilarity of section and variety of conditions forbid the use of formulas of this class.

4. The second class may be represented by Chezy's formula:

$$V = C \sqrt{rs.}$$

The particular form is of no consequence, for the criticism about to be

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* The strictness of the limitation is fully recognized by Capt. Cunningham in his formal statement: "Central mean velocity measurement appears to be the best means of rapid approximation to mean (sectional) velocity, but the reduction must at present be effected by a co-efficient to be found by previous special experiment at each site."

made is not as to co-efficient or exponent, but impeaches the terms employed, hydraulic mean depth, $r = \frac{\text{area}}{\text{wet perimeter,}}$ and sine of slope, $S = \frac{h-h'}{l}$.

The argument relied on is the following fact, furnished by the experiments of Mr. J. B. Francis at the Tremont weir and measuring flumes;* and illustrated by accompanying Diagram 1, Plate VII. The discharge was determined by careful observation at the weir. Above the weir the water passed through a flume of rectangular section. At the side of the flume a gauge was set from which the depth of water in the flume at each observation was read. The experiments cited were made to determine the formula of correction to be applied to vertical tube floats, and the facts were not only observed with care, but the suspicion of anything anomalous was carefully guarded against. After a somewhat extended series of observations, the flume being 26.745 feet wide, it was narrowed one-half or to 13.372 feet. In both cases the sides and bottom of flume were smooth planking. The conditions above and below the flume were unchanged.

In Diagram 1 the discharges by weir measurement are plotted as abscissas, to the corresponding depths in flume as ordinates. The full line curve presents the results of the wide flume observations, the broken line those of the narrow. It will be noticed that, for discharges greater than 250 cubic feet per second, the depths in flume were less after narrowing than before. It is certain that:

1st. The width being one-half and the depth for a given discharge somewhat diminished the sectional area was reduced to less than one-half its former value; therefore, the mean velocity must have more than doubled ($V' > 2V$).

2d. The surface elevation at the weir for a given discharge is a fixed level. A reduced depth in the flume means, therefore, a diminished slope from flume to weir, and local slope for any intermediate section must have been reduced ($S' < S$).

3d. The change in the section also effected a great reduction (30 per cent.) of hydraulic mean depth, therefore ($r' < r$).

In the formula

$$V = C\sqrt{rs.}$$

* See Table XXII, Lowell Hydraulic Experiments.

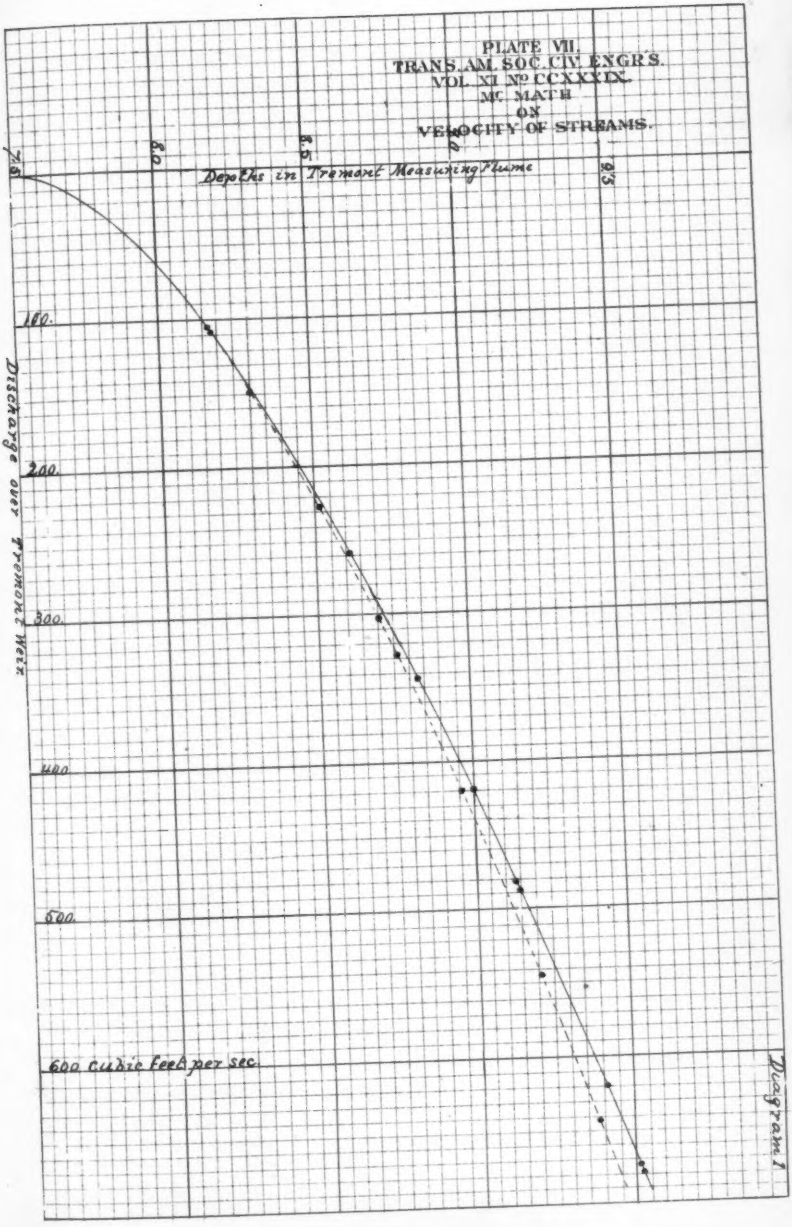
PLATE VII.
TRANS. AM. SOC. CIV. ENGRS.
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Mc MATH
ON
VELOCITY OF STREAMS.
Depths in Tremont Measuring Flume
7.5
8.0
8.5
9.0
9.3
Discharge over Tremont Weir
100.
200.
300.
400
500.
600 cubic feet per sec.
Diagram 1

We have shown a second state, V', r', and S', in which V is greatly increased while both of the variable terms in the second member have diminished. If the co-efficient C had been determined from the earlier observations it would utterly fail to satisfy the conditions of the changed section, though at the same site. If resort is made to a sliding scale of co-efficients, no material relief is afforded by D'Arcy and Bazin or Kutter. Their formulas would throw nearly the whole burden upon an increased slope somewhere. The facts show that no part of it was below the site.

5. The experienced engineer is at no loss to know that the water was driven through the measuring flume by a head which was chiefly concentrated as fall at the upper end of the flume. To that fall the term slope, as used by hydraulicians,

$$\left(S = \frac{h-h'}{l}\right)$$

is totally inapplicable. The sudden contraction caused an accumulation of head at the point of engorgement, under that head the velocity was accelerated, and the effect extended throughout the contracted channel, indeed, to the brink of the weir.* The head producing velocity, in this case, would clearly be measured by the height of surface, a little up stream from the point of contraction, above a horizontal plane through the crest of the weir. The inappropriateness of the term slope, applied to the energetic hydraulic factor, will be better apprehended if it be borne in mind that head is pressure, and cannot always be determined by the level. The pressure of the ocean tide forces the water in the Bay of Fundy to rise many feet above sea level, furnishing a clear example of the fact, by no means rare, that water does run up hill in open channels.

Surface slope may afford a measure of head in a strictly uniform channel, but can have no place in dealing with natural streams. Therefore, in the problem this paper discusses, all old formulas are entirely set aside.

6. Many engineers avoiding formulas have used diagrams of discharge, a curved line drawn through the points determined by plotting observed discharges as abscissas to gauge readings as ordinates, and by so doing have been able to interpolate discharges at dates and stages when direct measurements were not made.

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* There is reason to suspect that the slope from flume to weir crest became negative for the larger discharges after the flume was narrowed.

7. One need but consider attentively what such diagrams are, to gain a starting point for further study far in advance of Capt. Cunningham's suggested return to seeking a relation between mean and central velocity ; still further in advance of such as adhere to functions of slope and hydraulic mean depth. For the diagram of discharge, when capable of practical use (all are not), testifies to the fact that, under certain conditions, whose occurrence is not rare, there is but one independent variable, and that the readily observed element stage.

8. To reach discharge by Cunningham's method two elements must be observed for every desired result. Central velocity (which is, by his own showing, unreliable, unless a mean of many observations) to obtain mean velocity, and stage to obtain areas. The diagram requires the latter only. The diagram is local in its application, but no more so than the co-efficient of reduction, "to be found by previous special experiment at each site." Plainly, the preliminary observations will determine the diagram just as readily as they will the co-efficient.

9. Following this line of thought, it will be profitable to determine under what combination of conditions the simple relation indicated by certain discharge diagrams is realized, with the view of extending their intelligent use.* To facilitate the gauging of streams is one object, to guard the profession against the improper use of right methods is another. In pursuing these purposes results will be developed that have an important bearing upon some branches of engineering work.

10. The recurrence of fixed types of curve in discharge diagrams has already been alluded to, as conclusive evidence that but one independent variable enters into the problem of discharge.‡ This proposition is fundamental. The actual recurrence of types is a fact reached only by experience.

11. From such experience the curves have been formulated for the particular case of discharge over a weir. Weirs differ greatly in their conditions, but the discharge over each kind of weir has its law in terms of one variable ; height above crest of weir ; for conditions become constants, and formulas are general only as conditions are similar. If such

* Classification according to local conditions is the basis of D'Arcy and Bazin's categories and Kutter's diagram. Their principles of classification are too narrow. Material and smoothness of wetted surface do not avail to explain the facts cited from the Lowell experiments, for these were not changed.

‡ The reader will readily see that in rectangular and trapezoidal sections the variation of area may be directly expressed in terms of change of stage.

law exists at the weir, it must of necessity exist at every section above the weir and within its influence; for at all sections the discharge at any given time must be the same, and the local variations of stage must follow those near the weir by a close and intimate relation. Hence there must be a definite curve of discharge at each section when referred to local stage, under the condition that the sections undergo no change except a regular variation of area with stage.

12. Discharge diagrams have often been suggestive of a parabolic law, and particular equations have in some instances been determined. But, since discharges approaching zero are seldom observed (low water discharge is far from zero), there has been an acknowledged difficulty in locating the axis of the parabola, which logically must intersect the axis of ordinates at the level of no discharge. In a channel of continuous bed slope the level of no discharge is at channel bottom, and, as volume diminishes, flow ceases by extinguishment of depth when slope of stream surface is still considerable. Certainly a preferable condition for a discharge section would be a progressive diminution and simultaneous extinction of all the active elements of motion.

13. Taking the case of a stream above a weir or dam, without question the horizontal plane through the crest of the weir is the level of no discharge throughout the pool, above such weir or dam (See Fig. 1, Diagram 10). In natural streams the bars or shoals are substituted for the supposed dam, and we may state as a general proposition.

The plane of no discharge is determined by the crest of the bar or dam (of greatest height if there be more than one), and is limited by the intersection of the tangent plane with the bottom at or near some superior bar.

14. In a river this plane is the natural zero of hydraulic phenomena, and gauge readings might with propriety be referred to it, rather than to the imaginary standard of low water.

15. Persons using discharge diagrams have often found, when carrying the investigation farther, that by factorizing the discharge ($Q = V.A$) and plotting mean velocities and gauge readings as co-ordinates a series of points was developed whose mean could best be expressed by a straight line; that is, the relation between stage and mean velocity appeared to be a simple ratio.

If the discharge section be rectangular, and many sites are approximately so (considering only the part of section between high

and low stage), variation of area is also represented by a straight line.*

The curve of discharge being the product of two functions, which may be considered linear, will be a parabola, if both the functions are straight lines, but in no other case. I enlarge upon this point (for there is danger of taking too much for granted in practical use of the method). If the gauge reading referred to zero of discharge be represented by $\triangle$, and the section be a rectangle with bottom at zero, area (A) will be—

$$A = \triangle w,$$

in which, it will be remembered, width (w) is, by hypothesis, constant. Velocity is also supposed to be a straight line—

$$V = \triangle b.$$

The product is—

$$Q = A\,V = \triangle^2 b\,w,$$

whence $\triangle^2 = \frac{A}{b\,w} V,$

a common parabola, in which b is a constant to be determined. A discharge curve of this form would require three conditions : 1st. A rectangular section. 2d. That the section be empty when flow ceases. 3d. That the relation between stage and mean velocity be a simple ratio. A little consideration will enable the reader to satisfy himself that the second and third conditions are inconsistent, and the combination impossible.

16. If the zero of discharge is a plane above the bottom at the discharge section, there will be for that section an invariable area below the level of no discharge, which may be represented by a; its figure is immaterial. If the section above that level be rectangular, the area variable with stage will be the product of width into gauge reading ($\triangle w$). The total area is—

$$A = a + \triangle w.$$

Multiplying by $V = \triangle b$, the discharge is—

$$Q = A\,V = \triangle^2 b\,w + \triangle a\,b: \qquad (1)$$

whence—

$$\triangle^2 + \frac{a}{w}\triangle = \frac{A}{b\,w} V: \qquad (2)$$

but $\frac{a}{w}$ is the mean depth of the invariable part of the section, and $\frac{A}{w}$

* See Figs. 1 and 2, of Diagram 10.

is the mean depth of the total area, representing these mean depths by d and D, equation (2) becomes—

$$\triangle^2 + d. \triangle. = \frac{D}{b} V, \qquad (3)$$

under the conditions supposed, the formula for mean velocity would be—

$$V = \frac{b\,(\triangle^2 + d. \triangle)}{D}, \qquad (4)$$

which contains one co-efficient to be determined, and one variable to be observed.

17. The discharge curve for Section G, Connecticut river, shown by broken line of Diagram 6, is computed by an equation of the form (1)—

$$Q = 466 \triangle^2 + 5556 \triangle,$$

An alternate branch at the lower levels is added to the diagram to illustrate the risk of assuming lines to be straight from their origin, because a known part appears to be so. The true curve would lie between the alternates. At the higher levels the curve fits the observed discharges closely enough to show its practical value, and to warrant the position that, under the conditions named, the formulas just obtained are practically true for a considerable range of stage below the highest. In failure at low levels, it no more than follows in the track of the best weir formulas. For, at the weir, the discharge for small depths does not follow the law of relation to depth which is good at the higher levels, and I think the cause is the same.

18. It will be noticed that the formulas contain two terms not hitherto used in hydraulics : 1st. Permanent area (a), or that of the section below the level of no discharge. 2d. Ruling depth, defined to be the depth of the plane of no discharge below the surface, at any given time and place ; its symbol is $\triangle$.

19. Thus far the argument has accepted the apparent straightness of lines of mean velocity as real. But the plane of no discharge being now definitely located at the level of the weir or shoal, he who may have diagrams of discharge, if applying the test, will probably find that his curves do not intersect the axis of abscissas at the origin of co-ordinates, but behind it ; and the line of mean velocities, if it appears straight, will, when produced, cut the axis of ordinates above the origin. Minus velocities and discharges are absurd ; therefore we must conclude that the straightness of the line of mean velocities is deceptive, and it is really a curve whose origin is at the level of no discharge.

20. If mean velocities be a curve, then the curve of discharge is of a higher degree than the second, and it is well to turn from its study to the simpler curves of its factors. In so doing, variable area will still be considered rectangular, and discussion will be confined to mean velocity.

21. The fact that ruling depth, defined above, measures the phenomena has been sufficiently shown to justify its introduction as a substitute for hydraulic mean depth. It is a representative of surface slope, if one be needed, for if ruling depths at any two sections be taken, their difference is the fall of surface. Logically, it is entitled to the position of sole independent variable in a formula of mean velocity, for it measures head and the variations of area; therefore of both s and r.

22. If a stream be drained to its lowest limit, the surface in any pool will be a plane at the level of bar or weir crest. (In all strictness, the lowest point in the bar is the crest; but in practical application to streams of considerable size a mean level of crest may be used.) Velocity, head, and discharge will all be extinct; but at any given transverse section a certain area of motionless water may remain. This area has already been named permanent (for any section), and the symbol a assigned to it. If, now, the surface be raised at any section by $\triangle$, discharge begins, and mean velocity has a value which obviously must bear an inverse ratio to a. Increments of velocity, as $\triangle$ increases, will be so controlled by a if large, that the curve of velocity must be not only an orderly, but a very gentle curve.

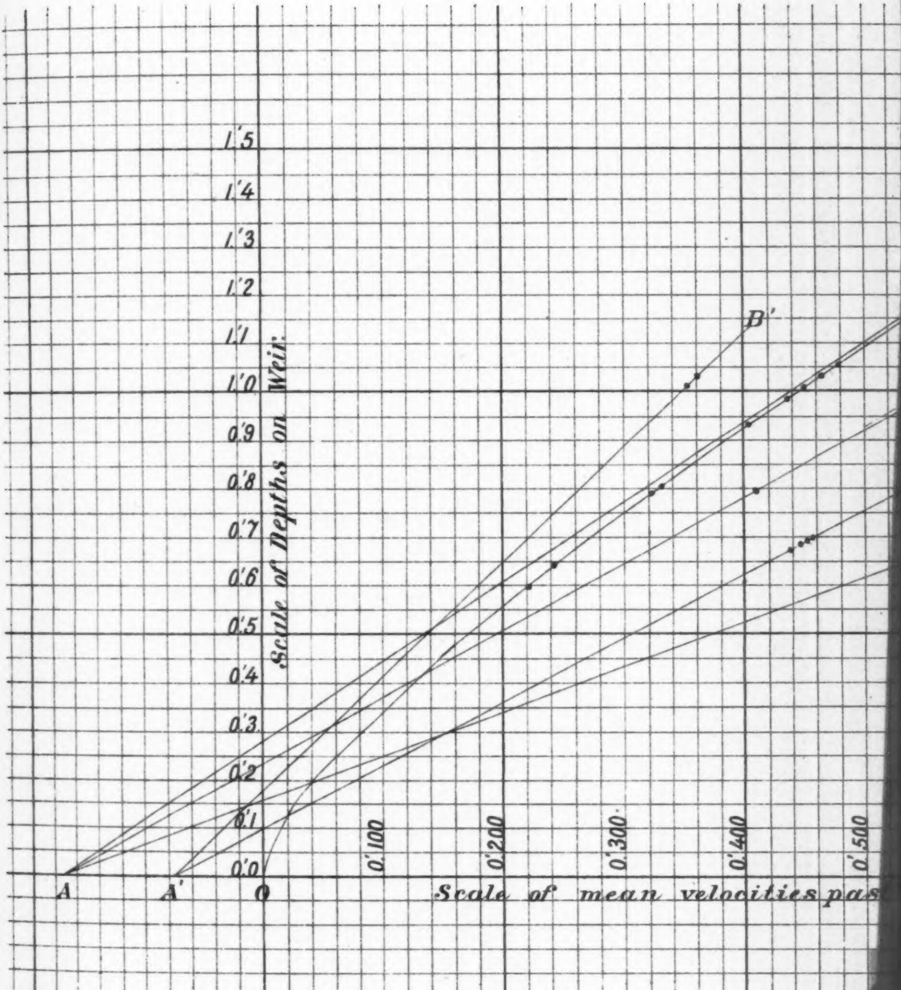
The cause of the apparent straightness of such curves is thus revealed, and we know that the one prime condition of a simple relation between V and $\triangle$ at any site is a large value of a.* On the other hand, an excessive value of a would render increments of V too small for accurate determination of discharge. Therefore, a mean value for permanent area should be sought at a discharge section.

23. The foregoing reasoning, and the evidence of observed velocity curves, which will shortly be considered, warrant the general proposition

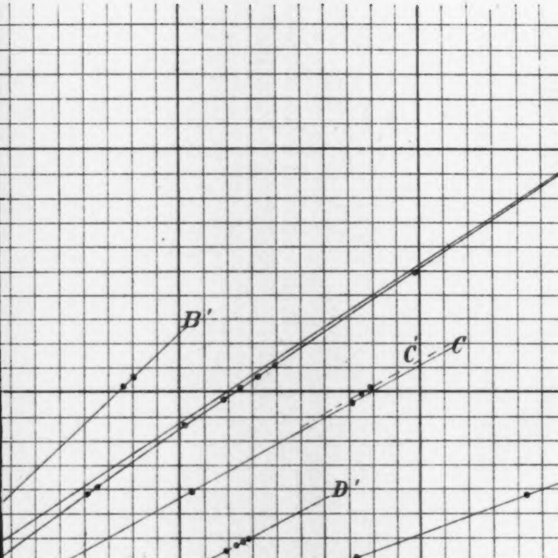
$$V = F. (a. \triangle).$$

This function, when developed, may be so complicated as to be useless; it may be simple in its particular form as applied to a given series of observations, and yet be a resultant of many complications; or it may, in its general form, be a simple expression containing constants appli-

* This prime condition of simplicity is directly contrary to Capt. Cunningham's conclusion that "Experimental sites should not be situated in marked hollows of the bed slope."



1.'5
1.'4
1.'3
1.'2
1.'1
1.'0
0.'9
0.'8
0.'7
0.'6
0.'5
0.'4
0.'3
0.'2
0.'1
0.'0
Scale of Depths on Weir.
B'
0.'100
0.'200
0.'300
0.'400
0.'500
A
A'
O
Scale of mean velocities pas


B'
C'
C
D'
0'300
0'400
0'500
0'600
0'700
mean velocities past section 6 feet

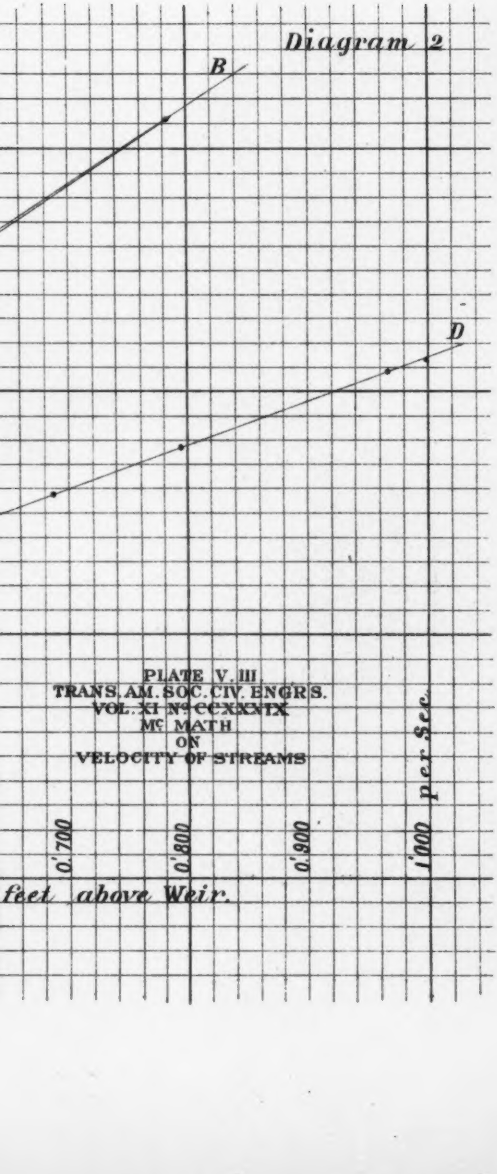


Diagram 2
B
D
PLATE V. III.
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0.700
0.800
0.900
1.000 per Sec.
feet above Weir.

cable to individual cases only. It is sufficient to claim, at this stage of the discussion, that both elements are readily observed.

24. A study of the accompanying diagrams is necessary to the further prosecution of the subject.

25. Diagram 1 has already been described. It is a discharge curve for the section at the gauge in the two conditions of that section.

Diagrams 2, 3 and 4 are representations of facts obtained directly, or legitimately derived, from Mr. Francis' Lowell Experiments. Diagram 2 being columns 7 and 8 of his Table XIII. Diagram 3 is obtained by computation from data given in Table XXII, and diagram 4 by a similar process from Table XVI.

The curved lines *O B*, of diagrams 2 and 4, are extensions of the observations by computing the discharges for heights not observed.* Observations, or the means of groups in most cases, are designated by dots. Lines curved or straight are added to interpret the observations.

Diagrams 5 and 6 are obtained from Gen. Theo. G. Ellis' observations upon the Connecticut river, near Thompsonville. Observations taken at the several sections are designated by enclosed dots. Values obtained by transfer of discharges are represented by simple dots. The chief uncertainty about such transfers is the local stage. This source of error is small between *F* and *G*, but may be material at *B* and *C*.

The remaining diagrams are not pertinent at present.

26. Diagram 2 contains five straight lines passing through, or near to, the observations. The line *A B* is tangent to the curve *O B* and passes through the upper pair of dots only. The extension of curve *O B* below the observations by computed discharges is mainly to locate the origin, and the origin so fixed may be taken as common to all velocity curves at the same locality, under like condition of weir height.

The observations shown along the curve *O B* were taken when the cross-section at the gauge station, 6 feet above the weir, was 13.96 feet wide and 5.048 feet deep below the level of the weir, less the area of the gauge boxes. The value of a was 70.02 square feet. Heights of surface at gauge above crest of weir ($\triangle$) are ordinates, and mean velocities past gauge are abscissas.

The incomplete curve *O C* results from a change of cross-section to 9.992 feet wide, and depth, as before, 5.048 feet. a was 50.44 square feet.

* These extensions are for illustration only, and are not referred to in the subsequent argument.

The incomplete curve $O\ D$ comes from a cross-section 13.96 feet wide and 2.014 deep below weir level, less the area of the gauge boxes, a being 27.67 square feet.

The straight lines $A\ C$ and $A\ D$ fairly satisfy the observations of the two series last named. The line $A\ B$ also satisfies the upper observations of the first series. The radiation of these lines from a common centre on the axis of abscissas will be noticed.

The two points located on the line $A'\ B'$ are observations taken with the cross section, as in the series $O\ B$, but the weir was divided into two equal bays by a partition.

The points on line $A'\ D'$ are observations with section, as in series $A\ D$, but the weir was divided as in $A'\ B'$. The convergence of these straight lines on axis of abscissas is again to be noted.

The two points on short broken line C' are observations under the same conditions as the series $A\ C$, except that the stream was prevented from spreading after passing the weir.

27. For observations not specially designed for the purpose, these of Mr. Francis at Lowell Lower Locks, illustrate the subject under discussion wonderfully. The variety of conditions and combinations is very nearly that required, and the exactness of observation is beyond the limit of expression by small scale diagrams.

The convergence of lines in the diagrams may be suggestive, but reference to figures will be instructive.

Computing the co-efficients of the straight lines passing through the upper groups of the three series OB, OC, and OD, I obtain

Straight line AB, $X = 0.61345\ (y - 0.2767) = 0.61345\ y - 0.1697$.
" " AC, $X = 0.72524\ (y - 0.2338) = 0.72524\ y - 0.1696$.
" " AD, $X = 1.09013\ (y - 0.1592) = 1.09013\ y - 0.1714$.
mean constant $-\ 0.1702$.

The convergence of the straight lines upon a point in the axis of abscissas, at a distance in rear of origin equal to 0'.1702, measured in velocity, is thus put beyond question.

Assuming the convergence to be at $-\ 0.170$, and computing the co-efficients that will best satisfy all the observations above a weir depth of 1 foot, by method of least squares, I obtain empiric formulas:

Series AB, $V = 0.6144 \triangle - 0.170$,
" AC, $V = 0.7253 \triangle - 0.170$,
" AD, $V = 1.0886 \triangle - 0.170$,

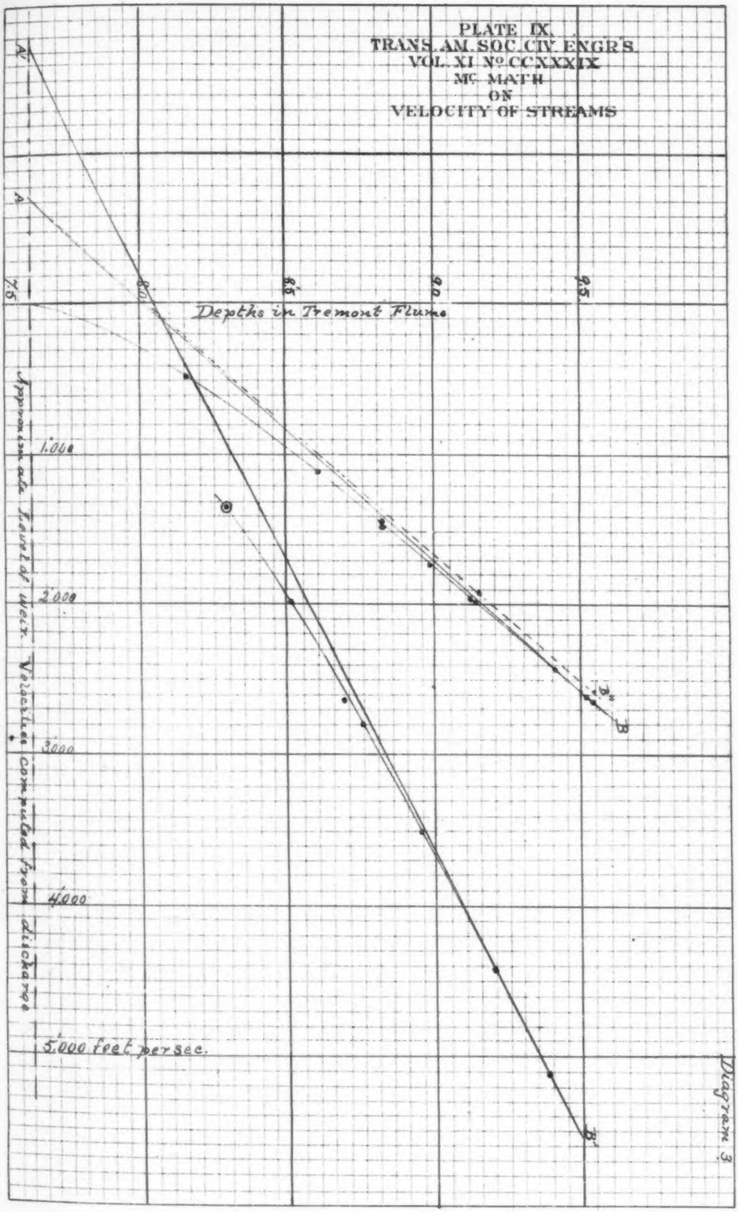
PLATE IX.
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Depths in Tremont Flume
7.5
8.0
8.5
9.0
9.5
Approximate Level of weir
Velocities computed from discharge
1.000
2.000
3000
4.000
5.000 feet per sec.
A'
A
B
B'
Diagram 3

these co-efficients differ but little from those previously obtained by taking the upper pair of observations in each series.

If we were to entertain the thought that these formulas are good at lower levels, we would meet the absurdity that flow over the weir must cease when the surface only 6 feet upstream was in

Series *AB*, 0.'2767 above weir level,
" *AC*, 0.2338 " " "
" *AD*, 0.1592 " " "

therefore a curved part must intervene between the lower observations of *AC* and *AD* and the origin, the same as shown by observation and computation for the series *OB*.

28. In the observations represented by diagram 4, the conditions were materially changed. The model of a dam was substituted for the weir, the new profile having a level crest 3½ feet wide, and a slope of 16½ feet, extending to the cross section at which the mean velocity is to be determined.

The discharge being actually measured in the lock chamber, mean velocity is easily found at any known section. The new section was 9.992 feet wide, and 5.85 feet deep below crest of dam (by scale on plan), whence a was 58.45 square feet.

From the experiments, a formula for the discharge over the dam was obtained by Mr. Francis,

$$Q = 3.01208lh^{1.53}$$

and from computed discharges the curve of mean velocity is extended to $\triangle = 3'.4$.

The curve passes into a line practically straight at $\triangle = 2'.5$. The equation of the secant line ($\triangle = 2.5$ to $\triangle = 3'.4$) is

$$V = 0.7258\ (\triangle - 0.461) = 0.7258\ \triangle - 0.3346.$$

The co-efficient is almost the same as for the line *AC* of diagram 2, and we are reminded that the widths of section are the same. The section in itself considered is as if the bottom of section in the former position had lowered 0'.8. In this view diagram 4 would, if the original weir had been retained, have given another line to diagram 2 intermediate to *AB* and *AC*. If we were to transfer it to diagram 2, the actual line would be parallel to *AC*, but originating at — 0'.3346.*

---

* It will be understood that the mode of obtaining the upper part of diagram 4 diminishes the value of its evidence, except as to general indications. No stress can be put upon parallelism to *AC*.

The shift of origin is by ready inference due to change in the form of weir (the level of crest was the same). Since the discharge for given heights above weir was lessened, the retraction of origin would appear to attend increased obstruction by the weir.

The division of the weir into two bays, in another case, was also an increase of obstruction, and we see by diagram 2, that the origin moved forward.

The series represented by the lines $A'B'$ and $A'D'$ are too short to be entirely conclusive, but it is hazarding little to say that both origin and inclination of the straight lines were changed by the division of the stream at the weir. Taking the figures for the two groups of observations represented by $A'B'$, and computing a line passing through them, I have

$$V = 0.431\ (\triangle - 0.192) = 0.431\ \triangle - 0.0827,$$

and for $A'D'$

$$V = 0.768\ (\triangle - 0.0988) = 0.768\ \triangle - 0.0759.$$

$$\text{mean constant} \qquad 0.0793.$$

The computed intersections with axis of abscissas, are very close together, and a trifling change would bring them to a common centre. We again have a practical convergence of secant lines for a given condition of weir and varying section.

Comparing the co-efficients of lines

$$AB \text{ and } A'B' \text{ we find } \frac{A'B'}{AB} = 0.7026$$

$$AD \text{ `` } A'D' \text{ `` `` } \frac{A'D'}{AD} = 0.7045$$

Wherefore the consequence of dividing the weir appears to have been a rotation of the lines AB and AD, without disturbing their relative inclination, and a movement of the point of convergence along the axis of abscissas. In Diagram 4 we have movement along the axis of abscissas, and probable rotation in the opposite direction to that in Diagram 2. The effect of dividing the stream is evidently much more serious than that of change in form of weir below level of no discharge.

29. Passing to Diagram 3, which represents experiments on a larger scale, and under conditions very unlike those of Diagrams 2 and 4, special note should be made of the fact that the discharge section is much farther from the weir, and the channel between spread out to three times the width at the flume. Therefore regularity cannot be attributed to the

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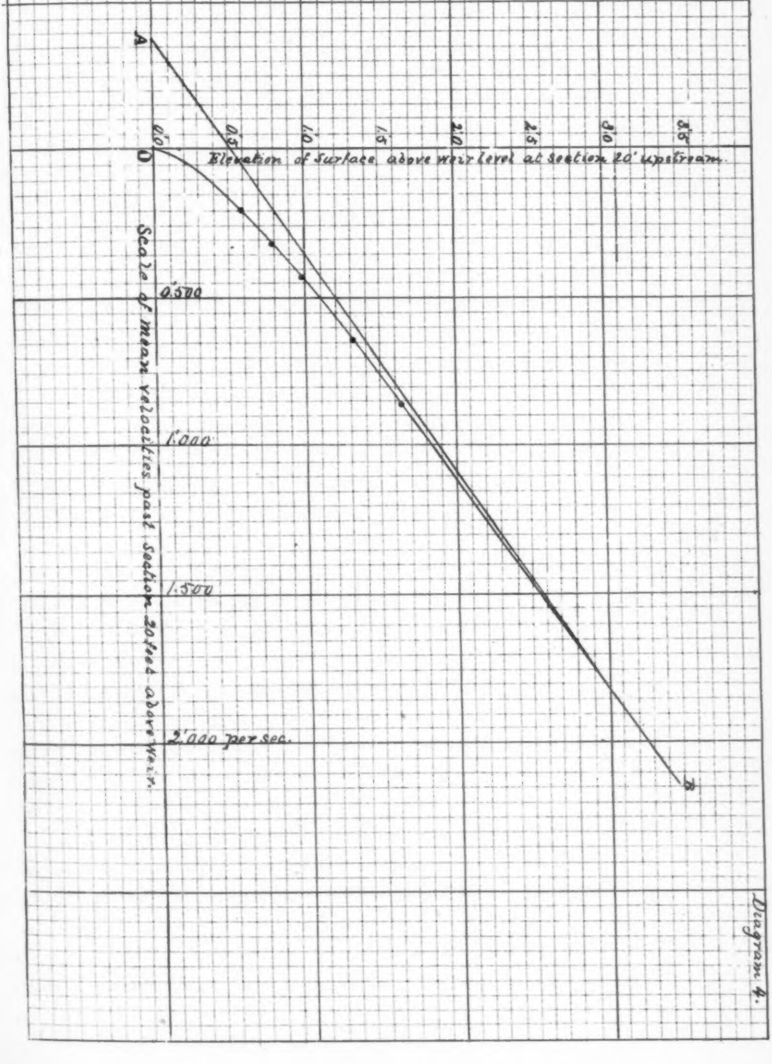

direct draft of the weir. The relation of weir level to the bottom of the flume is not known, but is not far from 7′.6. The velocity curves do not become straight lines, nor do the two secant lines approach a convergence at or near the axis. The weir condition was without variation, and, according to the precedent of the observations at the Locks, change in the section should not disturb the centre of the secants. This is a warning against hasty assumptions. We should notice that in this series of experiments the velocities were considerable, and the gauge located in a still water box, where it measured a column of water sustained by the pressure of the stream upon the sides of the flume, and not the level of the stream.

The diagrams have been made from the heights given in the tables, and for the practical purpose of this discussion the essential thing is shown by the observed heights, for the manifest simplicity and regularity of the velocity curve, with reference to stage, is seen not to depend upon theoretical niceties, but to belong to the observed values.

In diagram 3 a pair of points, each representing a group of six observations, require a distinct line AB''. These mark the effect of a disturbance of the conditions of approach to the gauge section, by which numerous whirls were caused, and these are seen to have diminished the capacity of the channel.

30. Summing up the positive teachings of the Lowell experiments, it is certain—

1st. There is a relation between mean velocity and ruling depth which is definite at any section under stable conditions.

2d. The relation at a given section depends upon the constancy of—

(*a*.) The section condition; its area must vary only with the stage, not by scour, or fill or artificial change of area or form.

(*b*.) The weir condition; its height, form and freedom of discharge must not vary.

(*c*.) Condition of approach; no changes in direction or regularity of flow.

31. To each of these three conditions the relation is sensitive to a degree that must satisfy the thoughtful Engineer of the hopelessness of obtaining a mean velocity formula with constants and co-efficients capable of extended use.

The possible combinations of conditions are infinite, but we have reason to assert that each combination affords a definite relation at

all stages, hence constants and co-efficients locally determined are reliable.

The third condition described must be, in effect, the same as increase of area below weir level, to diminish velocity for a given height of surface, and by so considering it the conditions (in kind) reduce to two, Section and Weir.

32. The weirs of Diagrams 2 and 3 are arranged for complete contraction. The dam section of diagram 4 gave a free overfall. The relation is not less definite in the one case than in the other.

Diagrams 5 and 6 introduce an ordinary dam in a large river. No one will expect in these diagrams the precision of results seen in Mr. Francis' experiments.

The height of dam is not stated, but its average was between 38 and 39 feet on the gauge. Sections *G* and *F* were at the ends of a short velocity base. *C* is some 3 800 feet nearer the dam, and *B* is an intermediate section.* *G F* and *C* have nearly the same area. *B* is the section of least area in the pool. Variable section is so nearly rectangular at each site that areas may be considered as straight lines.

The observed mean velocities arrange themselves upon a straight line, with some notable exceptions. The numbers attached to the diagram at *G′* show the order of observation. 1, 2, 27, 28 and 29 stand to the right of the straight line and testify to velocities greater than the mean. It is a singular feature that both groups 1, 2 and 27, 28, 29 are for a falling stage. The rising stage Nos. 24, 25, 26 are to left of line, and indicate velocities less than the mean for the stage, and very considerably less than prevailed at like stages during the subsequent decline, or just contrary to the rule that velocities are accelerated by rising and retarded during the falling stage. According to received ideas this is an anomaly. By the view here brought forward it is probable that, owing to the adjustment of supply and discharge, the river below the dam rose and fell relatively faster in this particular flood than in others ; that is, it is variation of the weir condition by back water during rise, and freer fall during decline. Nos. 35, 36 also stand apart from the lines indicated by the other observations. A canal capable of passing nearly the entire low water volume is reported to be fed from the pool ; its opening must be at a level considerably below the dam, and it is to be expected that the

* For details of sections see General Ellis' Report, in Rep. Chief of Engrs., 1878, Part 1, pages 351–353.

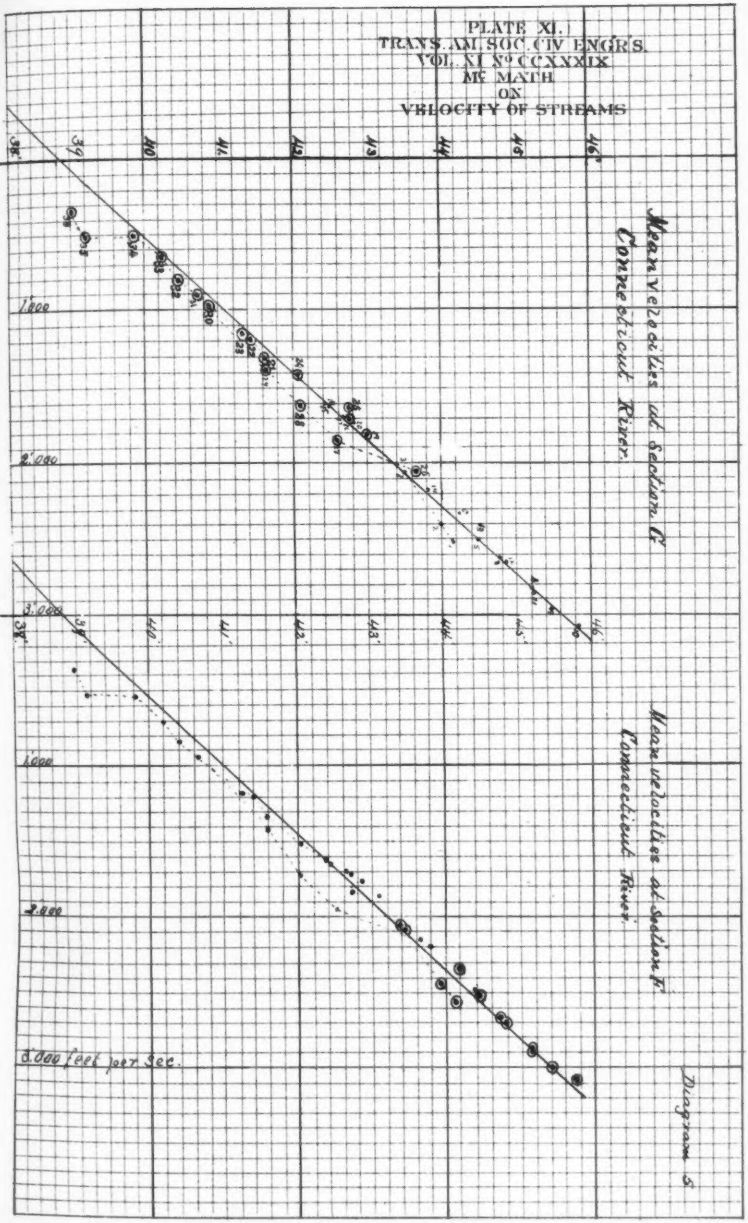
PLATE XI.
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Mean velocities at Section C
Connecticut River.
Mean velocities at Section F
Connecticut River.
Diagram 5.
3.000 feet per Sec.

control should pass to the lower opening, when the volume there escaping becomes relatively great compared with that passing over the dam. With these explanations the apparent anomalies support the law.

33. Uncertainty as to weir level prevents any attempt to trace relations or to determine the best line to satisfy the observations. That the line of Diagram *G* is near enough for practical purposes is evidenced by the resulting computed curve of discharge represented by the broken line of Diagram 6.

34. The diagram at section *B* has but three observed points, and they may be affected by weir variation ; but if they belong to the mean weir condition their teaching is important ; for from the direction of the connecting line it would intersect the axis of abscissas in front of origin. Therefore the curve must be tangent to axis of abscissas, whereas all other curves, that have come under notice, have been tangent to axis of ordinates. At section *B* the river is not only shoal, but much wider than at *G*, *F* or *C*, and the dissimilarity of velocity curves arises from the relations of widths and areas.*

35. Is has appeared in the course of this discussion—

1st. Negatively, that hydraulic mean depth and sine of slope do not measure the phenomena of water flowing in natural channels.

2d. Slope is a misnomer when applied to the distribution of fall and considered as the cause of motion.

3d. Positively, motion is due to head or pressure. Near a weir it is customary to measure head by taking the elevation of surface above crest of weir at a considerable distance from the crest. This distance being indefinite extends the relation of discharge to height above weir crest to all sections above the weir.

4th. The level of no discharge is the natural reference plane for hydraulic phenomena.

5th. Large sectional area below level of no discharge must, of necessity, limit the velocity to small and regular increments.

6th. Weirs, dams, and natural bars or shoals define the level of no

---

* If we carry the idea of diminished depth and area to its limit, it will appear that the locality must become itself a weir. Another case of tangency to axis of abscissas has been developed on the Mississippi below Memphis. This condition of the velocity curve indicates a section of undue width, and affords a criterion to determine the width of a regulated river. A large area and slack current at low stage is in the interest of navigation. Efficiency in discharge is a necessity in time of flood. Tangency to axis of ordinates meets both conditions. Tangency to axis of abscissas violates both.

discharge at all sections above their sites, until the horizontal plane through their crests intersects the bottom of the channel.

7th. Two new hydraulic terms are proposed—

(1.) Permanent area, or that part of transverse section below the level of no discharge.

(2.) Ruling depth, or the depth of the plane of no discharge below the surface at any time and place.

The first is a constant at a given section, but may differ materially at neighboring sections. The second is variable, and in rivers measures stage. For neighboring sections the differences will be the measure of surface slope. It also measures that part of area which varies with stage in all sections having regular side slopes.

8th. For a given site ruling depth is the only independent variable in the local mean velocity formula. The proof of this proposition rests upon the fact, that the best determined mean velocities, whether observed in natural or artificial channels, develop a smooth curve when plotted as abscissas to ruling depths as ordinates.

9th. Most of these curves become virtually straight lines at a moderate value of ruling depth.

10th. The curves, without impairing their simple regularity, are shown to vary with either of three conditions, namely:

(1.) That of weir, dam or bar, as freedom of discharge is increased or impaired.

(2.) That of the section, as its area is enlarged or diminished, other than by change of ruling depth.

(3.) That of approach, such as change in the direction of current, or in the irregularities of motion, whirls and eddies.

11th. The changes of curve caused by the 1st and 2d and 3d conditions are distinct and characteristic. Taking the direction of the upper parts of the velocity curves, after they become straight lines, and producing it backward to intersection with axis of abscissas, it is discovered that change of weir condition varies the point of intersection, and change of section condition varies the angle of intersection.

These eleven propositions are put forth as proven, or capable of proof from the facts given in the diagrams. I add two theoretical propositions, suggested by the preceding, but shall not discuss them.

1. The facts may be interpreted to signify that the velocity curves are

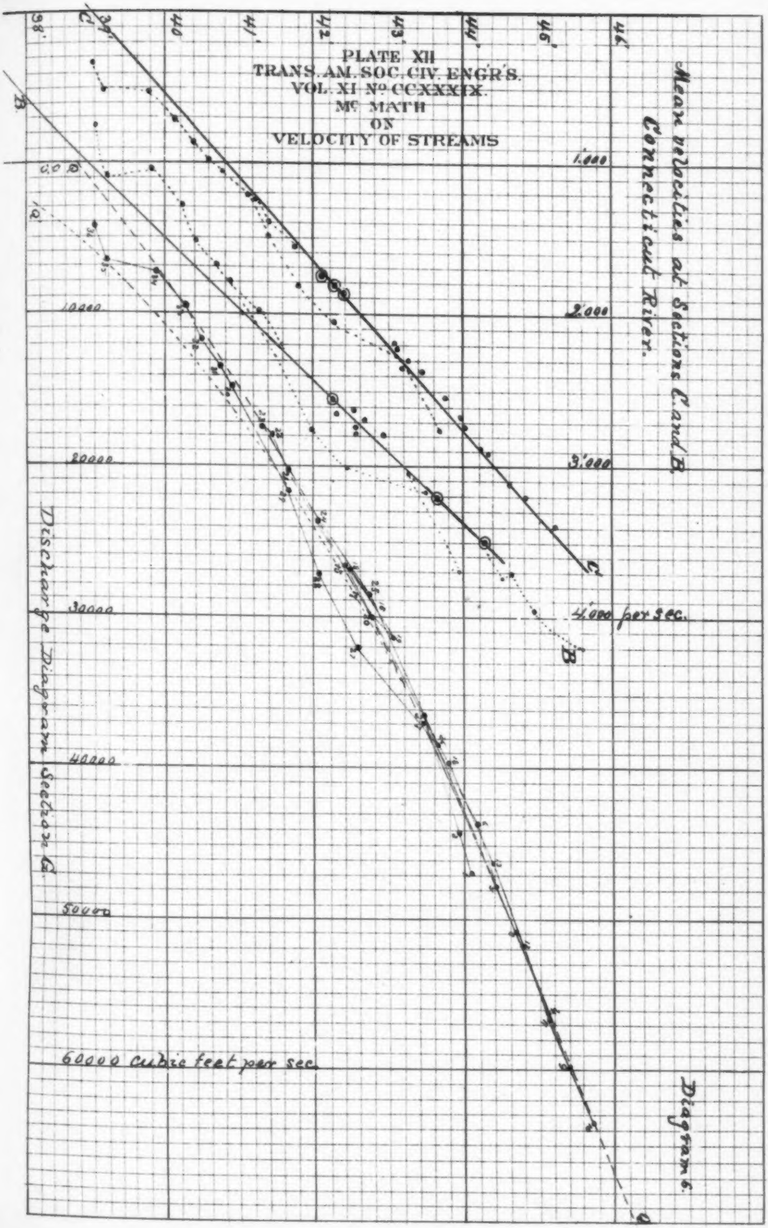

PLATE XII
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hyperbolas; weir condition being transverse, and section condition the conjugate axis.

2. The height of the cone from which the hyperbola is cut is a function of $2gh$ and area, h being the head available to produce motion at a given site. Each section furnishes its individual cone. See Fig. 3, Diagram 10.

36. In practical application of these propositions to gauging streams, everything is made to depend upon local determination of constants. Therefore it will be assumed that something more is desired than a single gauging.

Direct measurements of discharge by the best means and method available are essential. Let the local conditions be studied, that a few careful observations may give a local curve, which may be used with confidence so long as local conditions continue unchanged.

1. A desirable site for observation to obtain continuous discharge will not be found at a bar or wide section, but in the pool, at a point where the area and width have a mean value. If the permanent area be too great, the variation of velocity for the lower stages will be imperceptible; if it be too small, the law of the velocity curve will not appear, except by many observations covering the whole range of stage.

2. Sites should afford a distributed current, and in general should be taken in a straight reach as far from the preceding bend as possible.

3. The influence of tributaries must be considered; go below, if necessary, but never immediately above one. Back-water from this or any other cause is change of weir condition.

4. Choose a section with steep banks, and high enough to contain the stream at all stages. If this be impossible, be cautious of extending the curve observed below beyond the level of overflow; for overflow is an important change of section condition.

37. If these cautions are observed in choosing a site, a series of direct discharge measurements, including a considerable variation of stage, will furnish the direction of the curve of mean velocity after it has become practically a straight line. If the level of no discharge is also determined by observation of the controlling weir or bar, it becomes the axis of abscissas; heights above level of no discharge are ordinates. The steps required to determine the intersections of the straight line with the axes need not be specified.

The curve of mean velocity, if not a hyperbola, is very nearly one, and

may be computed from the data by considering the intersections with axes of abscissas and ordinates as defining the transverse axis (A), and the conjugate axis (B), respectively. The mean velocity curve referred to vertex of transverse axis is—

$$\triangle^2 = \frac{B^2}{A^2}\left(V^2 + 2AV\right) \qquad (5).$$

Theoretically, B and A are complex functions of a and $2gh$. But the practical process described will, if the conditions stated are observed, determine their local value. If thought advisable, mean velocities may be combined with areas, and a table or diagram of discharge prepared.

I add an additional caution. Choose times for preliminary discharge measurements when the stream is at a stand, if possible. If this is impracticable, obtain discharges at equal stages of rise and fall, and take a mean.

38. To use the method described, it is essential that stage, $\triangle$, should be observed at the discharge section, and that the section and regulating weir or bar be permanent.

39. In many natural streams both section and bar are unstable. So far from this working a suspension of the relations under discussion, it increases their importance. The suggestion, that the condition of the wier or bar exercised a controlling influence over velocity in the pool, came from a study of data observed upon the Mississippi. In these data the law of ruling depth was disclosed by its variations.

40. The purpose of the study was not so much to improve methods of gauging streams as to obtain light upon the important practical subject of flood control. The retention of flood waters within definite bounds manifestly depends upon the constancy of the relation between stage and volume ; or upon the expectation that an assumed, or ascertained, maximum volume will always discharge through a given cross-section.

41. Facts are on record showing that, at Columbus, Ky., in 1857 and 1858, the discharge of the Mississippi exceeded 1 100 000 cubic feet per second four times. At the first rise, December, 1857, the discharge named occurred at a stage of 29'.50 on the rise, and at 31'.60 on the decline, difference 2'.10. The second rise, in March, 1858, the stages for like discharge were 32'.50 and 34'.70, difference 2'.20. The third rise, April, 1858, the stages were 33'.50 and 35'.90, differcnce 2'.40. At the fourth rise, June, 1858, the stages were 35'.00 and 36'.90, difference 1'.90.

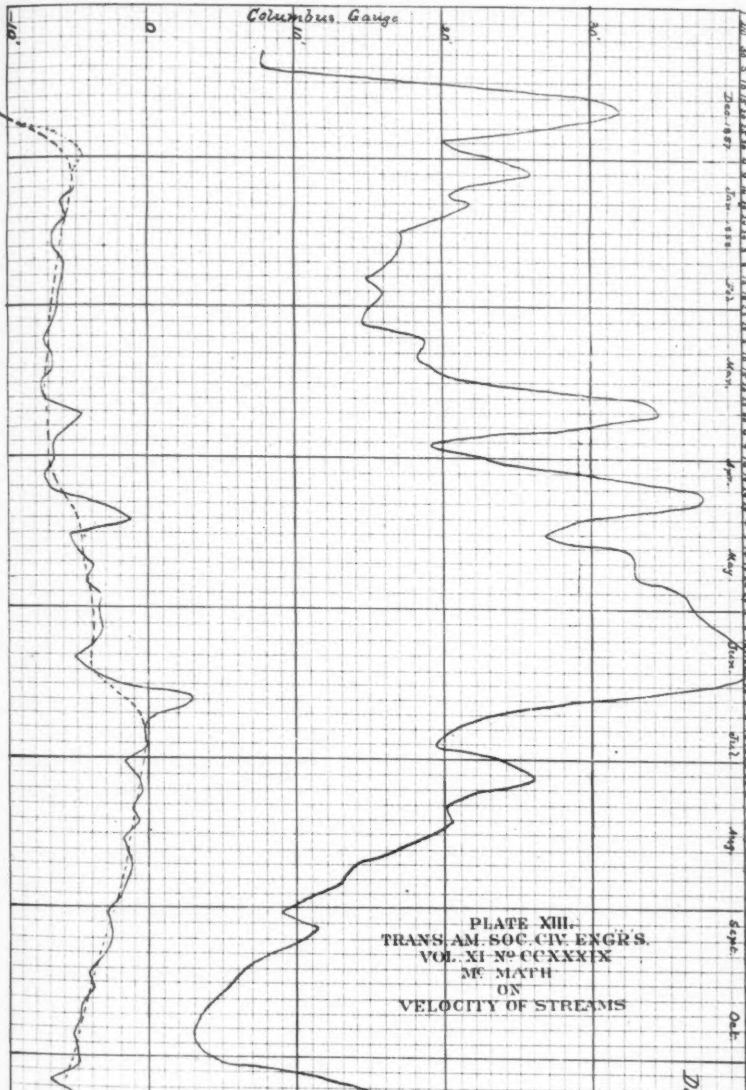

PLATE XIII.
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The mean of the differences, 2'15, is all that can in any case be accounted for by acceleration and retardation by stage.

The successive differences between stages of like discharge 3'.00 + 1'.00 + 1'.50 = 5'.50 on rise, and 3'.10 + 1'.20 + 1'.00 = 5'.30 on decline are evidence of the progressive impairment of the capacity for discharge. The mean of the sum of these differences, 5'.4, suggests that overflow by the flood of 1858 was not due wholly to volume, but in a considerable part to an unknown cause, which diminished the capacity of the channel between December, 1857, and June, 1858.

42. In a paper read at the 12th Annual Convention of this Society (No. CCVI, vol. 9, of Transactions), the writer discussed the variation of the section of rivers and showed, that a filling of wide sections at times of flood was to be expected in silt conveying streams, and conversely a removal of deposit during low stages*. If this fill be in the channel it is a building up of the shoal, and the subsequent scour is a lowering.

43. In the present paper I have shown the importance of weir conditions by undisputable facts, and have, by natural inference, attributed like consequences to a change of dam, bar or shoal. There is a wide difference between a weir, discharging into open air, and a submerged dam or bar. I hoped that Captain Cunningham's observations might furnish facts that would definitely establish the inference : for the Ganges Canal is regulated by dams, which are varied at will and therefore afford many phases of weir condition. Unfortunately the "state of control" is given by Cunningham only as a ratio of closed area to that of a full opening. I therefore must fall back upon inferior evidence.

44. The diagrams of Humphreys and Abbot (Plates XIV, XV, XVI and XVII of their report), are discharge diagrams, but bear slight resemblance to the diagrams 1 and 6 accompanying this paper. If mean velocities were substituted for volumes, the resulting diagrams would present an intricacy like to the discharge. Yet out of the apparent confusion order will come, if the disturbing effect of three causes be allowed for.

First. Velocities are accelerated by a rising and retarded during a falling stage. Rise and return to a stand at a given stage would produce a looped curve of velocity. The Columbus observations furnish four

* A fuller discussion of the same subject will be found in Appendix K, and confirmatory facts in Appendix D, to Report of Mississippi Commission for 1881.

distinct loops : the breadth of any one loop is much less than the breadth occupied by all, and the influence of this cause may be to that of all causes, as breadth of any one loop is to the breadth of belt included between extremes. If the figures be taken for each rise, as it passed a thirty foot stage, the breadth of 1st loop was 0'.66, of 2d, 0'.52, of 3d, 1'.08, of 4th, 1'.42 feet per second. Total breadth between extremes being 3 feet per second. This cause, therefore cannot be credited with more than $\frac{7}{15}$ of total effect, and probably this is much too great, for the progressive lessening of velocity between first and fourth rise must in part have occurred between the dates embraced in each loop, consequently the general cause has also widened the loops.

Second. Change within the section by deposit or erosion. This may have been :

1st. An enlargement under the high velocities of the flood stage, 7½ to 8½ feet per second.

2d. A deposit in the slack current of the low stage, 1½ to 2 feet per second.

3d. The irregular fleeting changes due to passage of sand waves.*

The first two would be most active in producing irregularity at extremes, and the results would be progressive during continuance of high and low velocity. Since, by remeasurement of section after the flood had passed, enlargement by several thousand square feet was observed, the broadening of the upper parts of the loops may have been due to this cause, but, since the observed enlargement was much greater at the lower of two sections, 200 feet apart, than at the upper, it is probable that the apparent change is due to a sand wave. Sand wave effects would compensate each other within a few days.

Third. Variation of level of no discharge, chiefly by fill and scour at the regulating shoal situated at an unknown distance below the observation station.

Experience has taught navigators and engineers that the channel depth over a Mississippi bar is subject to change of several feet by fill during high and scour at low stages, its effect as weir variation should be correspondingly great.

45. At Columbus the several loops furnish a straight line of velocity with a constant co-efficient, but variable origin.

* The subject of sand waves has only received attention during the last few years. See Appendix D, to Rep. Mississippi River Commission.

PLATE XIV
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Scale of depths below 0 of gauge
25'
20'
15'
10'
5.0'
0'
30'
40'
Feb. 1859
Mar.
Apr.
May
Jun.
Jul.
Aug.
Sept.
Oct.
Nov.
Vicksburg Gauge
0.
10'
20'
30'
40'
Diagram 8.

The mean of all lines is

$$V = 0.9212 + 0.1849\,y, \text{ or practically}$$

$$V = 0.185\,(y + 5'),$$

the limits of extremes being $(y + 11.1)$ and $(y - 3'.0)$: y is the reading of the gauge, whose zero was 5'.7 above low water of 1855.

Variable origin of the straight line has, in the earlier part of this paper been identified as effect of change of weir condition. The present purpose is to see if change of origin measures variation of bar height in a river of unstable regime. The weight of the test depends wholly upon the probability of the results, that is their accordance in season, progress and amount, with what are now known to be general facts.

Dividing each observed velocity by 0.185, and subtracting the corresponding gauge reading from the quotient, I obtain the positions at which the straight line would intersect the gauge at each date, and have plotted the results as lower line of Diagram 7; the upper line being stage of water. The change of bar height cannot be sudden, and I have drawn a broken line as a mean, to express the probable effect of weir condition, allowing the first two causes, named above, to have produced the serrations shown by the full line. It is assumed that the vertical shift of the straight line along the gauge reflects, if it does not measure, the change in level of no discharge.

46. By a similar process applied to the Vicksburg observations of 1858, I obtain Diagram 8. The mean position of line is :

$$V = 0.097\,(y + 22.7). \quad \text{Limits } y + {20'.0 \atop 27'.9}$$

47. Similarily I obtain from the Carrollton observations of 1851, Diagram 9, mean position of line being :

$$V = 0.324\,(y + 4.'0). \quad \text{Limits } y + {2'.2 \atop 5'.5}$$

48. The three diagrams concur in showing that the cause of velocity variation is associated with the greater or season oscillations of stage, having a minimum at low water, and a maximum following the culmination of the flood at a considerable interval. The lesser maxima and minima follow the minor flood waves at Columbus and Vicksburg.

49. These diagrams show the cause—varying as my former paper showed that bar height must vary. And the amount of variation of the lower lines of the three diagrams is in accord with the known changes of Mississippi bars. Identification in the absence of direct observation could scarcely be more complete.

50. Carrying the teachings of the weirs to the broader case of river bars,* is now justified by the regularity and constancy of the relation between mean velocity and ruling depth when the latter is variable. The facts at hand furnish one more indirect, but, when apprehended, convincing proof.

The observations at Carrollton, which furnish data for Diagram 9, were 123 in number, and made at "Prime Base." At intermediate dates 8 similar observations were made at "Preston Base," about a mile up-stream; 3 at "Race Course Base," 4 500 feet down stream; and 4 at "Locks Base," 9 000 feet below "Prime Base." Diagram 9 furnishes a measure of the variations of the level of no discharge at "Prime Base," and it is certain, if truth has been reached in the preceding discussion, that these variations must be the same at all the sites.

If for any observation the mean velocity be divided by the vertical distance between the line of stage and the broken lower line of Diagram 9, for the corresponding date, the quotient will be the co-efficient of the straight line for the site, and for any site the co-efficient should have a constant value. (The influence of the other two causes of velocity variation will cause the co-efficients so obtained to vary somewhat.) Comparing the co-efficients determined for different sites they should approximate an inverse ratio to the areas at the several sites. The following table exhibits the tests:

* In the original study the order was reversed. Weirs confirmed conclusions reached from discussion of river data.

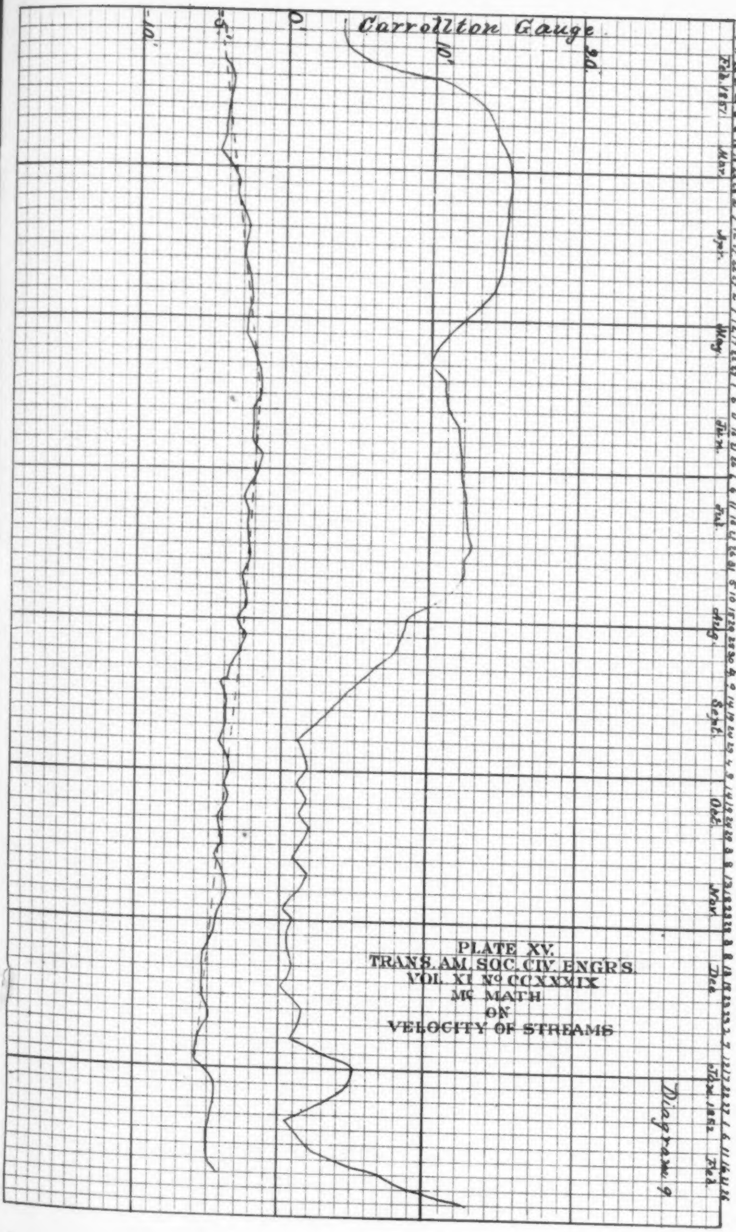

PLATE XV.
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Site and Date. "Preston Base."	Stage.	From Diagram, △-Stage.	△	V	Co-efficient. $= \frac{V}{\triangle}$	Ratios
Apr. 9, 1851....	15.0	3.10	18.10	4.84	0.2674	Co-ef. $\frac{\text{Prime Base}}{\text{Preston Base}} =$
" 10, "	15.1	3.00	18.10	4.70	0.2595	$\frac{0.324}{0.264} = 1.227$
" 11, "	15.0	3.00	18.00	4.64	0.2578	Low water area $\frac{\text{Preston B.}}{\text{Prime B.}}$
" 16, "	14.8	2.90	17.70	4.75	0.2683	$\frac{191\ 045}{150\ 840} = 1.267$
" 18, "	14.7	2.80	17.50	4.68	0.2674	High water areas $\frac{\text{Preston B.}}{\text{Prime B.}}$
May 12, "	12.3	2.30	14.60	3.87	0.2651	$\frac{219\ 540}{179\ 210} = 1.225$
June 2, "	10.9	1.80	12.70	3.41	0.2685	
Oct. 20, "	1.7	3.80	5.50	1.42	0.2582	
Mean	....	....	....	......	0.2640	Co-ef., 1.227. Areas, 1.246
"Locks Base."						
May 13, 1851....	12.1	2.30	14.40	4.19	0.2909	Co-ef. $\frac{\text{Prime B.}}{\text{Locks B.}} = \frac{0.324}{0.283} = 1.145$
" 20, " ...	10.6	2.10	12.70	3.66	0.2890	L. W. areas $\frac{\text{Locks B.}}{\text{Prime B.}} =$
June 4, "	11.1	1.70	12.80	3.64	0.2844	$\frac{172\ 058}{150\ 840} = 1.141$. H. W. areas
Oct. 21, "	1.7	3.80	5.50	1.47	0.2673	$= \frac{203\ 170}{179\ 210} = 1.134$
Mean....	....	....	....	......	0.2829	Co-ef., 1.145. Areas, 1.138
"Race Course."						
May 15, 1851....	11.5	2.20	13.70	4.50	0.3285	Co-ef. $\frac{\text{Prime B.}}{\text{Race Course}}\ \frac{0.324}{0.327} = 0.991$
" 23, "	10.1	2.00	12.10	3.94	0.3256	L. W. A's $\frac{147\ 103}{150\ 840} = 0.975$
June 3, "	11.0	1.70	12.70	4.16	0.3276	H. W. A's $\frac{178\ 111}{179\ 210} = 0.994$
Mean	....	....	....	......	0.3272	Co.ef. 0.991. Areas, 0.985

The computed co-efficients are fairly constant at each site, and the means approximate the inverse ratio of areas. The conclusion is legiti-

mate that a common cause equally affected each of the four sites at any given date, but in a degree which varied with the season.

51. Returning to "Prime Base," the recorded data show that the mean depth (below a datum plane), of the upper section increased by 6'.76 feet between February and September, and the mean depth of the lower section, 200 feet distant, by 6'.82 feet. The indicated change of level of no discharge, comparing February and September, was about one foot rise. Hence the local change in section cannot be the cause of the variation under investigation, because such change could not have equally affected all the sites at a given time.

52. The bar at the mouth of the river, in the light of the facts before us, must have acted as a dam submerged in back water, the level of no discharge being intermediate to crest of bar and Gulf level. The amount of variation shown in Diagram 9 and the times of maxima and minima agree closely with known variations of depth at the passes.

The Mississippi, Connecticut and Mr. Francis' flumes present the same facts. In the case of the flumes, the weir or dam is the manifest cause of variation in level of no discharge. The Connecticut dam, and the Mississippi shoals are causes of like variation.

53. Aside from the general confirmation of a definite relation between mean velocity and ruling depth, the facts last presented show that, having stage and mean velocity, we can measure the changes in what I have called the weir condition. By inversion of process, if stage and weir condition are given by observation, and the local co-efficient determined, mean velocity and discharge may be easily and closely estimated by the method already described.

54. In a discussion which, though confined to old data, leads to the rejection of several notions that have almost attained the standing of accepted truths, and which brings forward new terms to express a novel conception of hydraulic relations, I cannot expect to convince, except the reader studies, digests and repeats. Still less can I expect to have avoided obscurity in expression, or misinterpretation of facts. But I do hope, that discussion and study will lead to experiment in the direction herein suggested, and experiment to better theory and practice.

PLATE XVI
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